Calculus III, Test 3 Review

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•The test will cover sections 13.9, 13.10, and 14.1-14.8.

•To study, you can look over your notes, rework HW problems on WebAssign, quizzes, and problems from the notes, as well as work out the practice problems given for each section. The Review Questions at the end of Chapters 13 and 14 are also good practice, as is the Test 3 from last year which is posted on my website.

•Calculators are allowed on this test, but for certain questions you may not be allowed to use a calculator. It is highly recommended that you bring a calculator because you cannot use cell phones or computers during most of the test. I have not decided yet if there will be a question where you can use a computer.

•Note that although most of the formulas are given to you on the formula sheet, there are some that are not- for example, you need to know the formula for surface area from section 14.5.

•Some practice problems to work on are below. Note that for some of the integrals, it may be necessary to change the limits of integration or switch to polar, cylindrical, or spherical coordinates

1. The paint for a room costs \$0.05 per square foot for ceiling paint and \$0.12 for paint for the walls (no paint on the floor). If the room has volume of 1000 ft^3, what are the dimensions for the room that will minimize the cost of the paint? What will that minimum cost be?

2. Find the maximum production level P if the total cost of labor (at \$72 per unit) and capital (at \$60 per unit) is limited to \$250,000, where x is the number of units of labor and y is the number of units of capital and $P(x, y) = 100x^{0.4}y^{0.6}$.

3. Evaluate the iterated integral
$$\int_0^4 \int_{\sqrt{x}}^2 \frac{3}{2+y^3}\, dy\, dx.$$

4. Find the volume of the solid formed by the intersection of the surfaces $z = 1 - xy$, $y = 1$, $y = x$, $z = 0$, and $x = 0$. You can use Sage/Maple/Grapher to help you figure out what it looks like if you need to.

5. Evaluate the integral:
$$\int_0^3 \int_0^{\sqrt{9-x^2}} (x^2 + y^2)^{3/2}\, dy\, dx$$

6. Find the center of mass of the lamina bounded by $y = 4 - x^2$ and $y = 0$, with a density function of $\rho = ky$.

7. Write a double integral that represents the surface area of $f(x, y) = x^2 + y^2$ over the region $R : \{(x, y) : 0 \leq f(x, y) \leq 16\}$. Integrate by hand if you can, and if you cannot explain why and find the integral using a computer algebra system (like Sage/Maple).

8. Sketch the solid whose volume is given by the iterated integral $\displaystyle\int_0^4 \int_0^{(4-x)/2} \int_0^{(12-3x-6y)/4} dz\, dy\, dx$. Then rewrite the integral to have order of integration $dz\, dx\, dy$, and compute whichever integral you think is easier.

9. For the triple integral

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{16-x^2-y^2}} \sqrt{x^2+y^2}\, dz\, dy\, dx,$$

convert from rectangular to both cylindrical and spherical coordinates. If one conversion is particularly annoying/nasty, you can also just explain why it is bad. Choose the easiest method and do the integration.

10. Use a change of variables to find the solid region lying below the surface

$$f(x, y) = (x+y)^2 \sin^2(x-y)$$

and above the region R which is bounded by the square with vertices $(\pi, 0)$, $(3\pi/2, \pi/2)$, (π, π), $(\pi/2, \pi/2)$.